A Second-Order Microwave Differentiator

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***Abstract*—A novel approach consisting of discrete signal processing (DSP) technique and optimization method is developed to implement a second-order differentiator at microwave frequencies. To utilize the transfer function of a second-order differentiator developed in DSP study, we formulate the chain scattering parameters of transmission lines in the Z domain. In particular, it is shown that shunt stubs combining with nonuniform serial lines lead to the realization of a second-order microwave differentiator. Some examples are presented to illustrate the validity of this approach.**

***Index Terms*—Nonuniform line, second-order differentiator, Z transform.**

I. INTRODUCTION

THE DIFFERENTIATOR is a very useful tool to determine and estimate time derivatives of a signal. It has been used extensively in many areas, such as image process, speech system and digital control. In radars and sonars, the velocity and acceleration of objects are computed from position measurements using differentiators [1]. In biomedical engineering applications, it is often necessary to attain the higher order derivatives of biomedical data.

Various methods had been developed to design discrete finite impulsive response (FIR) and infinite impulsive response (IIR) differentiators [2]–[5]. Al-Alaoui [2] used interpolation method to develop a stable, minimum-phase digital differentiator. Pei and Shyu [3] used the eigenapproach to design high order digital differentiators. In order to obtain low relative error, Kumar and Ohba [4] employed optimal method to develop digital differentiators which are maximally accurate at low frequencies.

Most differentiator studies thus far had been limited to DSP investigation for applications in the low frequency range. Thus, the implementation of differentiators for high-frequency applications has been largely ignored. In this letter, we employ both Z-domain method and equal-length transmission lines to implement a second-order differentiator having an operating frequency bandwidth of several gigahertzs.

II. THEORY

The transfer function of a stable first-order differentiator [2] in the Z domain is given by

$$G(z) = \frac{0.43(1 - z^{-2})}{(1 + 0.34z^{-1})^2}. \qquad (1)$$

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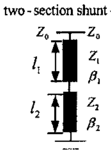
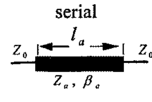
TABLE I
CHAIN-SCATTERING-PARAMETERS OF TRANSMISSION LINES

	$\begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}_z$	
two - section shunt - short 	$\frac{1}{E(z)}\begin{bmatrix} \frac{Z_0F(z)+2Z_1E(z)}{2Z_1} & \frac{Z_0F(z)}{2Z_1} \\ -\frac{Z_0F(z)}{2Z_1} & \frac{-Z_0F(z)+2Z_1E(z)}{2Z_1} \end{bmatrix}$	where $E(z)=1-z^{-2}$, $F(z)=1-2\gamma z^{-1}+z^{-2}$, $\gamma=\frac{Z_2-Z_1}{Z_2+Z_1}$
serial 	$\frac{1}{z^{-1/2}(1-\Gamma^2)}\begin{bmatrix} 1-\Gamma^2z^{-1} & -(\Gamma-\Gamma z^{-1}) \\ \Gamma-\Gamma z^{-1} & -\Gamma^2+z^{-1} \end{bmatrix}$	where $\Gamma=\frac{Z_a-Z_0}{Z_a+Z_0}$

$G(z)$ in (1) is obtained by inverting the transfer function of an interpolating integration rule. The interpolating integration rule is obtained by taking 0.7 Simpson rule and 0.3 trapezoidal rule. $G(z)$ has a better relative error than the differentiator based on Simpson integration rule. At 0.6 of full-band normalized frequency, $G(z)$ has a relative error of 3.2%, which corresponds to -30 dB. In particular, $G(z)$ has excellent linearity performance in the lower frequency range. The linearity of $G(z)$ degrades gradually as the frequency increases. $G(z)$ deviates from the characteristic of an ideal differentiator significantly for the normalized frequency greater than 0.7. By taking the square of $G(z)$, we obtain the transfer function of a second-order differentiator, as follows:

$$H(z) = \frac{0.43^2(1 - z^{-2})^2}{(1 + 0.34z^{-1})^4}. \qquad (2)$$

The frequency-domain responses of $H(z)$ and $G(z)$ are obtained by setting $z = e^{j2\omega\tau}$, where ω is the angular frequency and τ is the sampling time interval.

Table I shows the chain-scattering-parameters (or T) matrices of a serial transmission line and a two-section shunt-short stub in the Z domain, where β_i ($i = a, 1, 2$), Z_i, and l_i are propagation constant, characteristic impedance, and physical length, respectively. Note that Z_0 is the reference characteristic impedance, which is assumed to be 50 Ω, unless mentioned otherwise. We assume that all finite lines have the same electrical length, i.e., $\beta_a l_a = \beta_1 l_1 = \beta_2 l_2 = \omega\tau$, where τ is the propagation delay time of finite lines. The Z-domain chain scattering parameters of transmission lines are obtained by setting $z^{-1} = e^{-2j\beta_i l_i}$.

If a circuit is composed of M serial transmission-line sections and K two-section shunt-short stubs (M and K are positive integers), the overall chain scattering parameters $T_{11,\,overall}(z)$ of such a circuit is obtained by the sequential multiplication of chain-scattering-parameters matrix of each individual transmission element [6]. We then have

$$T_{11,\,overall}(z) = \frac{\sum_{i=0}^{M+2K} b_i z^{-i}}{(1-z^{-2})^K \prod_{m=1}^{M} \left(z^{-1/2}(1-\Gamma_m^2)\right)} \tag{3}$$

where all b_i are real and are determined by the characteristic impedances of all transmission lines. If the output of the circuit is loaded with a matched termination, the transfer function of the overall circuit, denoted as $S_{21}(z)$, is equal to $1/T_{11,\,overall}(z)$. We then have

$$\begin{aligned} S_{21}(z) &= \frac{1}{T_{11,\,overall}(z)} \\ &= z^{-M/2} \frac{(1-z^{-2})^K}{\sum_{i=0}^{M+2K} B_i z^{-i}} \end{aligned} \tag{4}$$

where $B_i = b_i / \prod_{m=1}^{M} (1-\Gamma_m^2)$ is a function of characteristic impedances of transmission lines in either serial or shunt configurations. The term $(1-z^{-2})$ in the numerator of (4) is due to a two-section shunt-short stub, and the term $z^{-M/2}$ represents the delay factor of serial transmission lines.

A closer examination of (2) and (4) indicates that we may use M serial lines and two two-section shunt-short stubs to emulate a second-order differentiator. Neglecting the propagation delay factor $z^{-M/2}$ in (4), we obtain the following:

$$\frac{0.43^2(1-z^{-2})^2}{(1+0.34z^{-1})^4} \approx \frac{(1-z^{-2})^K}{\sum_{i=0}^{2K+M} B_i z^{-i}}. \tag{5}$$

Note that B_i in (5) is determined by the characteristic impedances of all transmission lines. Upon using the optimization method [6] in the sense of least-mean-square error for the denominators in (5), we obtain the characteristic impedances of transmission lines.

III. Experimental Results

To construct a microwave differentiator, we employ microstrips to emulate transmission lines. Fig. 1 shows the physical layout of the microstrips which is built on a Duriod substrate having thickness 31 mil (0.78 mm) and relative dielectric constant $\epsilon_r = 2.5$. Here we use a five-section ($M = 5$) serial line shunted with two two-section shunt-short stubs ($K = 2$). The characteristic impedances of transmission lines are obtained by using an optimization process [6] that involves the comparison between two equivalent autoregression (AR) processes representing both $H(z)$ and $S_{21}(z)$. To assure that the values of characteristic impedances are practically realizable, the lower and upper bounds of characteristic impedances for the optimization procedure is set to be $5.0\ \Omega \le Z_i \le 155.0\ \Omega$.

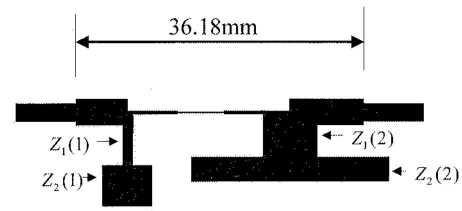


Fig. 1. Physical layout of the microstrips for a second-order differentiator.

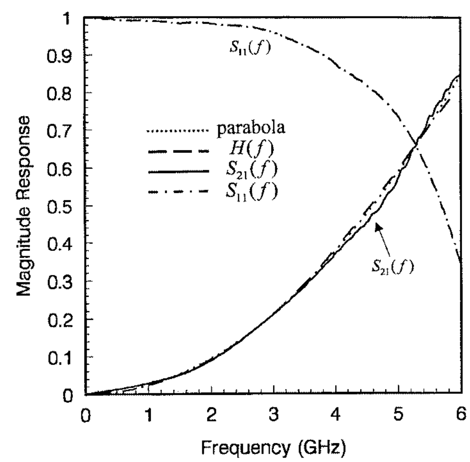


Fig. 2. Magnitude responses of the differentiator. $H(f)$ is the theoretical value and $S_{21}(f)$ is the measured result for the circuit in Fig. 1. $S_{11}(f)$ is the measured reflection coefficient. The magnitude response of an ideal differentiator is also shown, of which it varies as the square of signal frequencies.

The characteristic impedances of serial lines from the left side to right side are 38.4 Ω, 120.3 Ω, 150.9 Ω, 120.2 Ω, and 38.6 Ω, and the characteristic impedances of shunt stubs are $Z_1(1) = 69.5\ \Omega$, $Z_2(1) = 23.7\ \Omega$, $Z_1(2) = 22.0\ \Omega$, and $Z_2(2) = 7.1\ \Omega$. Here, the reference impedance for both input and output ports is 50 Ω. The ideal propagation delay time τ of each finite line is 25 ps which is corresponding to a maximum operating frequency of 10 GHz. The 25 ps delay time is equal to the propagation delay of a wave propagating through a quarter wavelength of a 10 GHz signal. To count for the discontinuity effect at each junction, we adjust the physical length of each finite line [7]. Therefore, the final propagation delay time of each finite line may not be exactly 25 ps. The ground termination of shunted stubs is implemented by using through-hole vias. Fig. 2 shows the theoretical values as well as experimental results of the differentiator for the frequencies extending from dc to 6 GHz. We should notice that 6 GHz represents 0.6 of full band normalized frequency. Apparently, the measured frequency-domain scattering parameter $S_{21}(f)$ is in good agreement with $H(f)$. For convenience, Fig. 2 also shows the magnitude response of an ideal second-order differentiator which varies as the square of signal frequencies. As stated previously, $H(f)$ in (2) deviates from the characteristic of an ideal differentiator significantly for the frequencies greater than 0.6 of full band normalized frequency. Therefore, the frequency responses of the differentiator in that portion are omitted. For convenience, Fig. 2 also shows the measured reflection coefficient $S_{11}(f)$ of the second-order differentiator shown in Fig. 1.

IV. Conclusion

A second-order microwave differentiator was implemented by using microstrip transmission lines. In particular, the Z-domain formulations of scattering characteristics of nonuniform transmission lines facilitate the implementation of discrete-domain differentiator in microwave circuits. It is plausible that many other circuits developed in DSP study can also be implemented by using nonuniform transmission lines for microwave applications.

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